1m

Charpit's Method

When the given equation canot be ruduced to any of the standered forms, then this methed is applicable to solve such equations.

Let us given eq^n be

$$f(x, y, z, p, q) = 0 \quad \text{— (1)}$$

If Possible Let another parallel relation be found as

$$F(x, y, z, p, q) = 0 \quad \text{— (2)}$$

Differenceating (1) & (2) w.r.to x, weget

$$\frac{\partial f}{\partial x} + 0 + \frac{\partial f}{\partial z} \cdot p + \frac{\partial f}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial x} = 0$$

& $$\frac{\partial F}{\partial x} + 0 + \frac{\partial F}{\partial z} p + \frac{\partial F}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial F}{\partial q} \cdot \frac{\partial q}{\partial x} = 0$$

eliminating $\frac{\partial p}{\partial x}$, we get

$$\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial z} p + \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial x}\right) \frac{\partial F}{\partial p} = \left(\frac{\partial F}{\partial x} + \frac{\partial F}{\partial z} p \cancel{+ \frac{\partial F}{\partial q} \cdot \frac{\partial q}{\partial x}}\right.$$
$$\left. + \frac{\partial F}{\partial q} \cdot \frac{\partial q}{\partial x}\right) \cdot \frac{\partial f}{\partial p}$$

or, $$\frac{\partial f}{\partial x} \cdot \frac{\partial F}{\partial p} + \frac{\partial f}{\partial z} \cdot p \frac{\partial F}{\partial p} + \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial x} \cdot \frac{\partial F}{\partial p}$$
$$= \frac{\partial F}{\partial x} \cdot \frac{\partial f}{\partial p} + \frac{\partial F}{\partial z} p \cdot \frac{\partial f}{\partial p} + \frac{\partial F}{\partial q} \cdot \frac{\partial q}{\partial x} \cdot \frac{\partial f}{\partial p}$$

$$\text{or, } \left(\frac{\partial f}{\partial x}\cdot\frac{\partial F}{\partial p} - \frac{\partial F}{\partial x}\cdot\frac{\partial f}{\partial p}\right) + p\left(\frac{\partial f}{\partial z}\cdot\frac{\partial F}{\partial p} - \frac{\partial F}{\partial z}\cdot\frac{\partial f}{\partial p}\right)$$

$$+ \frac{\partial q}{\partial x}\left(\frac{\partial f}{\partial p}\cdot\frac{\partial F}{\partial q} - \frac{\partial F}{\partial p}\cdot\frac{\partial F}{\partial q}\right) = 0$$

similarly differenceating ① & ② w.r.to y, weget

$$\left(\frac{\partial f}{\partial y}\cdot\frac{\partial F}{\partial q} - \frac{\partial F}{\partial y}\cdot\frac{\partial f}{\partial q}\right) + q\left(\frac{\partial f}{\partial z}\cdot\frac{\partial F}{\partial q} - \frac{\partial F}{\partial z}\cdot\frac{\partial f}{\partial q}\right)$$

$$+ \frac{\partial p}{\partial y}\left(\frac{\partial f}{\partial p}\cdot\frac{\partial F}{\partial q} - \frac{\partial F}{\partial p}\cdot\frac{\partial f}{\partial q}\right) = 0$$

But as $\frac{\partial p}{\partial y} = \frac{\partial^2 z}{\partial x\cdot\partial y} = \frac{\partial^2 z}{\partial y\cdot\partial x} = \frac{\partial q}{\partial x}$

Hence eleminating $\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$ weget

$$\frac{\frac{\partial f}{\partial x}\cdot\frac{\partial F}{\partial p} - \frac{\partial F}{\partial x}\cdot\frac{\partial f}{\partial p} + p\left(\frac{\partial f}{\partial z}\cdot\frac{\partial F}{\partial p} - \frac{\partial F}{\partial z}\cdot\frac{\partial f}{\partial p}\right)}{\frac{\partial f}{\partial q}\cdot\frac{\partial F}{\partial p} - \frac{\partial F}{\partial q}\cdot\frac{\partial f}{\partial p}}$$

$$= \frac{\frac{\partial f}{\partial y}\cdot\frac{\partial F}{\partial q} - \frac{\partial F}{\partial y}\cdot\frac{\partial f}{\partial q} + q\left(\frac{\partial f}{\partial z}\cdot\frac{\partial F}{\partial q} - \frac{\partial F}{\partial z}\cdot\frac{\partial f}{\partial q}\right)}{\frac{\partial f}{\partial p}\cdot\frac{\partial F}{\partial q} - \frac{\partial F}{\partial p}\cdot\frac{\partial f}{\partial q}}$$

$$\text{or, } \left[\frac{\partial f}{\partial x}\cdot\frac{\partial F}{\partial p} - \frac{\partial F}{\partial x}\cdot\frac{\partial f}{\partial p} + p\left(\frac{\partial f}{\partial z}\cdot\frac{\partial F}{\partial p} - \frac{\partial F}{\partial z}\cdot\frac{\partial f}{\partial p}\right)\right]\cancel{\left(\frac{\partial f}{\partial p}\cdot\frac{\partial F}{\partial q} - \frac{\partial F}{\partial p}\cdot\frac{\partial f}{\partial q}\right)}$$

$$= \left[\left(\frac{\partial f}{\partial y}\cdot\frac{\partial F}{\partial q} - \frac{\partial F}{\partial y}\cdot\frac{\partial f}{\partial q} + q\left(\frac{\partial F}{\partial q}\cdot\frac{\partial f}{\partial z} - \frac{\partial f}{\partial q}\cdot\frac{\partial F}{\partial q}\right)\right]\cancel{\left(\frac{\partial f}{\partial q}\cdot\frac{\partial F}{\partial p} - \frac{\partial F}{\partial q}\cdot\frac{\partial f}{\partial p}\right)}$$

$\left(\text{i.e. } \frac{\partial f}{\partial p} \cdot \frac{\partial z}{\partial q} \right)$

$$\text{or, } \left(\frac{\partial f}{\partial x} + p\cdot\frac{\partial f}{\partial z}\right)\frac{\partial F}{\partial p} + \left(\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}\right)\cdot\frac{\partial F}{\partial q}$$

$$\left(-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}\right)\cdot\frac{\partial F}{\partial z} + \left(-\frac{\partial f}{\partial p}\right)\frac{\partial F}{\partial x}$$

$$+\left(-\frac{\partial f}{\partial q}\right)\cdot\frac{\partial F}{\partial y} = 0 \quad -(3)$$

Which is linear equation of the order one with x, y, z, p, q as independent variables and F behaves as dependent variable

Here the charpit's auxilarly eq^n are

$$\frac{dp}{\frac{\partial f}{\partial x} + p\cdot\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}}$$

$$= \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dF}{0} \quad -(4)$$

Any integral of (4) will satisfy (3)

putting the value of p, q in this equation $dz = p\,dx + q\,dy$, which on integration gives the sol^n.

(8) Charpits method solve.

$$z^2(p^2z^2+q^2)=1 \quad -(1)$$

sol^n: $\therefore p^2z^4+q^2z^2-1=0=f(x,y,z,p,q)$

$\therefore$ charpits A.E is.

$$\frac{\partial p}{\frac{\partial f}{\partial x}+p\frac{\partial f}{\partial z}}=\frac{\partial q}{\frac{\partial f}{\partial y}+q\frac{\partial f}{\partial z}}=\frac{dz}{-p\frac{\partial f}{\partial p}-q\frac{\partial f}{\partial q}}=\frac{dx}{-\frac{\partial f}{\partial q}}=\frac{dy}{-\frac{\partial f}{\partial q}}=\frac{\partial f}{0}$$

$$\therefore \frac{dp}{0+p(4p^2z^3+2q^2z)}=\frac{dq}{q(4p^2z^3+2q^2z)}=\frac{dz}{-p(2pz^4)-q(2qz^2)}$$

$$=\frac{dx}{-2pz^4}=\frac{dy}{-2qz^2}=\frac{\partial F}{0}$$

taking first we we get

$$\frac{dp}{p(4p^2z^3+\cancel{2q^2z})}=\frac{dq}{q(4p^2z^3+\cancel{2q^2z})}$$

$$\therefore \frac{dp}{p}=\frac{dq}{q} \Rightarrow \log p=\log q+\lg c$$

$\therefore p=aq$

putting in (1)

$$z^2(a^2z^2q^2+q^2)=1$$

$$q^2=\frac{1}{z^2(a^2z^2+1)} \text{ or, } q=\frac{1}{z\sqrt{a^2z^2+1}}$$

or $p=\dfrac{a}{z\sqrt{a^2z^2+1}}$

subsitituing $dz=p\,dx+q\,dy$

$$\therefore dz=\frac{a}{z\sqrt{a^2z^2+1}}dx+\frac{1}{z\sqrt{a^2z^2+1}}dy$$

$\therefore z\sqrt{a^2z^2+1}\, dz = a\,dx + dy$

put $a^2z^2+1 = t^2$

$\therefore 2a^2z\,dz = 2t\,dt$

$\Rightarrow a^2z\,dz = t\,dt \quad \Rightarrow z\,dz = \frac{t}{a^2}\,dt$

$\therefore \frac{t^2}{a^2}\,dt = a\,dx + dy$

$\therefore \frac{1}{3a^2}t^3 = \frac{1}{3a^2}(a^2z^2+1)^{3/2} = ax+y+b$

(8) solve $\quad p = (z+qy)^2 \quad -①$

Sol^n → Here $f(x,y,z,p,q) = -p+(z+qy)^2 = 0$

$\therefore$ charpits A.E is

$$\frac{\partial p}{\frac{\partial f}{\partial x}+p\frac{\partial f}{\partial z}} = \frac{\partial q}{\frac{\partial f}{\partial y}+q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p}-q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}}$$

or, $$\frac{\partial p}{2p(qy+z)} = \frac{\partial q}{4q(qy+z)} = \frac{dz}{(-p)(-1)-q\cdot 2(qy+z)y}$$

$$= \frac{dx}{-(-1)} = \frac{dy}{-2y(qy+z)} = \frac{\partial F}{0}$$

Taking $\frac{dp}{2p(qy+z)} = \frac{dy}{-2y(qy+z)}$

$\therefore \frac{dp}{p} = \frac{dy}{-y}$

$\Rightarrow \frac{dp}{p} + \frac{dy}{y} = 0$

$\Rightarrow \log p + \log y = \log a$

or, $p = \frac{a}{y}$

From ① $p = (z+qy)^2$

$\Rightarrow \frac{a}{y} = (qy+z)^2$

$\Rightarrow \sqrt{\frac{a}{y}} = qy + z$

$\therefore q = \frac{\sqrt{a}}{y^{3/2}} - \frac{z}{y}$

$\because dz = p\,dx + q\,dy$

$\therefore dz \;\; \frac{a}{y}\,dx + \left(\frac{\sqrt{a}}{y^{3/2}} - \frac{z}{y}\right) dy$

$\Rightarrow y\,dz + z\,dy = a\,dx + \frac{\sqrt{a}}{\sqrt{y}}\,dy$

integrating

$\Rightarrow$

$\Rightarrow yz = ax + \sqrt{ay} + b$